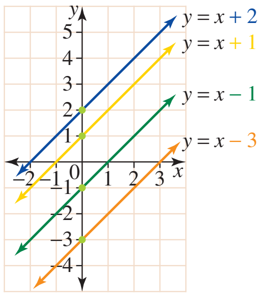
3-B

Identifying features of linear equations

KEY CONCEPTS

The *y*-intercept

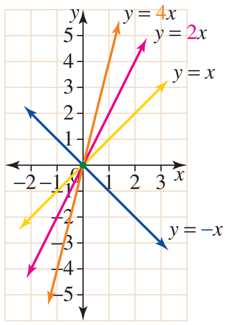


- The graph at right shows a number of straight lines. Each line has the same slope but passes through the y-axis at a different point.
- The **y-intercept** is the point at which a line crosses the y-axis.

Line colour	Equation	Cuts y-axis at:	y-intercept
Blue	$y = x + 2$	2	(0, 2)
Yellow	$y = x + 1$	1	(0, 1)
Green	$y = x - 1$	$^{-}1$	(0, $^{-}1$)
Orange	$y = x - 3$	$^{-}3$	(0, $^{-}3$)

- The constant in the equation and the y-intercept of the line are related.

Lines with different slopes

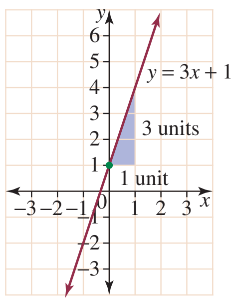


- Some lines may appear as if they are going uphill, while others will head downhill. The graph at right shows a number of straight lines with the same y-intercept but different slopes.
- The coefficient of x in the equation and the slope of the line are related.

Line colour	Equation	Slope
Orange	$y = 4x$	4
Pink	$y = 2x$	2
Yellow	$y = x$	1
Blue	$y = {}^{-}x$	$^{-}1$

- The slope or **gradient** of a line can be determined by measuring the change in the y-value for each increase of 1 unit in the x-value.
- The change in the y-value is referred to as the **rise**. A negative value for the rise indicates that the y-value is decreasing.
- The increase in the x-value is referred to as the **run**.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$



- If a right-angled triangle is formed (using the line itself as the hypotenuse), the 'rise'(vertical distance) and 'run' (horizontal distance) can be found.
- The gradient (m) of the line can be calculated by dividing the rise by the run.

$$m = \frac{\text{rise}}{\text{run}}$$

- In the diagram shown at right, the line has a rise of 3 units and a run of 1 unit.

Therefore the slope $= \frac{\text{rise}}{\text{run}}$
$= \frac{3}{1}$
$= 3.$

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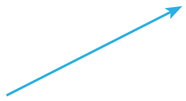
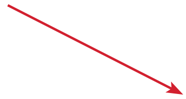
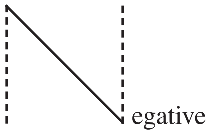
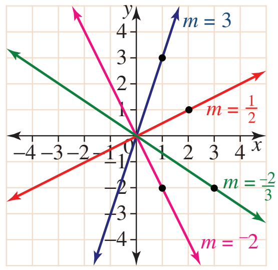
			
A positive gradient has a positive slope.	A negative gradient has a negative slope.	A horizontal line has no slope and a gradient of zero.	A vertical line has an infinite slope and an infinite or undefined gradient.

- A negative slope follows the same slope as the diagonal line in a capital letter N.



- The greater a slope's magnitude (positive or negative value), the steeper the line formed.

m	$m = \frac{\text{rise}}{\text{run}}$	Vertical distance	Horizontal distance
3	$\frac{\text{rise}}{\text{run}} = \frac{3}{2}$	Rise of 3	Run of 1
$\frac{1}{2}$	$\frac{\text{rise}}{\text{run}} = \frac{1}{2}$	Rise of 1	Run of 2
$\frac{^-2}{3}$	$\frac{\text{rise}}{\text{run}} = \frac{^-2}{3}$	Fall of 2	Run of 3
$^-2$	$\frac{\text{rise}}{\text{run}} = \frac{^-2}{1}$	Fall of 2	Run of 1



EXAMPLE 1

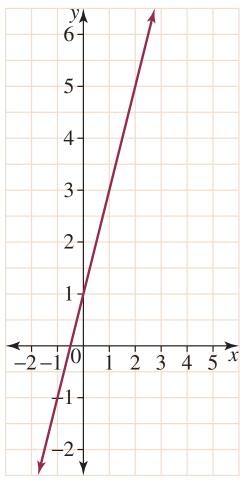
For each of the graphs shown:

i state the y-intercept for the line

ii determine the gradient of the line

iii describe the graph.

a



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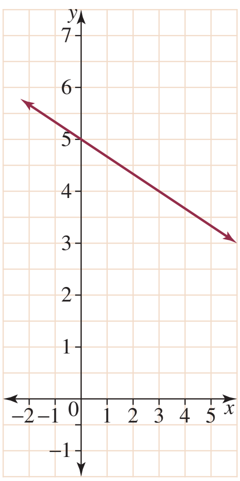


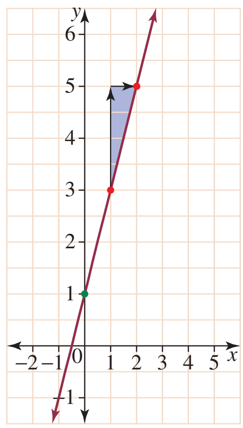
THINK

WRITE

a **i** The y-intercept is the point at which the line passes through the y-axis. → The y-intercept is (0, 1).

ii ❶ Gradient = $\frac{\text{rise}}{\text{run}}$.

- Look for two points where the x- and y-coordinates can be easily read from the grid, as shown in red.
- Form a right-angled triangle and use this to find rise and run.
  - The y-value increases from 3 to 5, a rise of 2.
  - The x-value increases from 1 to 2, a run of 1.



❷ Write the gradient formula and calculate the gradient. →

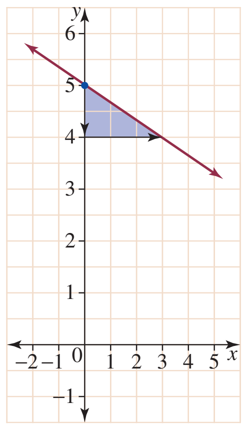
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{1}$$
$$= 2$$

iii Use the shape of the line, the y-intercept and gradient to describe the graph. → The graph is a straight line with a y-intercept of (0, 1) and a gradient of 2.

b **i** The y-intercept is the point at which the line passes through the y-axis. → y-intercept is the point (0, 5)

ii ❶ • Look for two points where the x- and y-coordinates can be easily read from the grid. →

- Form a right-angled triangle and use this to find rise and run.
  - The y-value decreases from 5 to 4, a rise of $^{-}1$.
  - The x-value increases from 0 to 3, a run of 3.



❷ Write the gradient formula and calculate the gradient. →

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{^{-}1}{3}$$

b iii Use the shape of the line, the y-intercept and gradient to describe the graph. → The graph is a straight line with a y-intercept of (0, 5) and a gradient of $\frac{^{-}1}{3}$.

LEARNING EXPERIENCE

WHOLE CLASS

Linear graphs puzzle

Equipment: strips of overhead transparency paper with a long straight line printed on each, thumbtacks, base plates (e.g. thick cardboard), BLM doc-0108 *Linear graphs puzzle sheet*

1 The thumbtack is to act as a *y*-intercept for each linear relationship. Place the thumbtack through the printed line on the clear strip (about halfway along).

2 Each linear relationship is described by a *y*-intercept and gradient. Place the thumbtack through the worksheet (using the base plate to protect your working surface) so that it corresponds to the *y*-intercept given for the first linear relationship.

3 Rotate the strip so that the line matches the given corresponding slope.

4 Record the letter that the line passes through.

5 Repeat steps 2 to 5 for the remainder of the linear relationships represented.

6 The letters spell out an important question. What is the question?

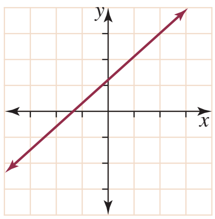
7 As a class, discuss how the *y*-intercept and gradient alter the position of the line on the graph.

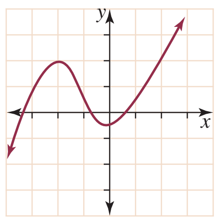
EXERCISE 3B

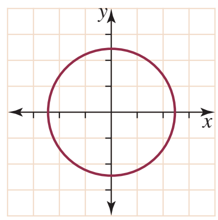
Now try these

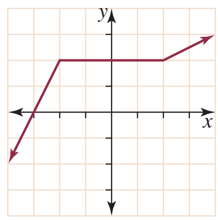
1 Describe the general shape formed by a linear equation.

2 MULTIPLE CHOICE Which of the following graphs represents a linear equation?

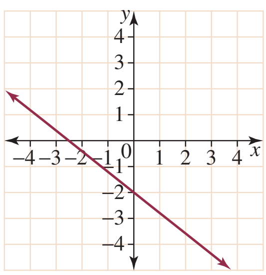
A


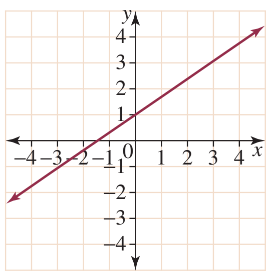
B


C


D


3 Each graph below represents a linear equation. State the *y*-intercept for each line.

a


b


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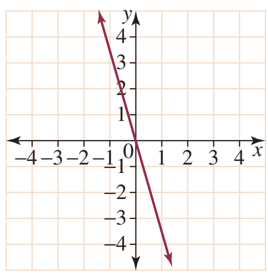
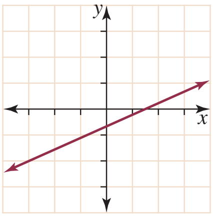


4 For each of the linear graphs below, state whether the gradient is positive, negative or zero.

a



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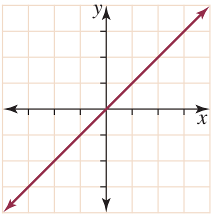

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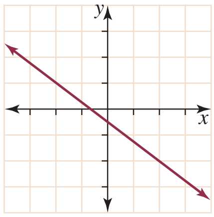

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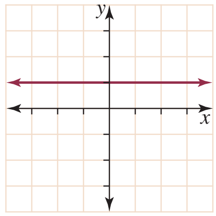
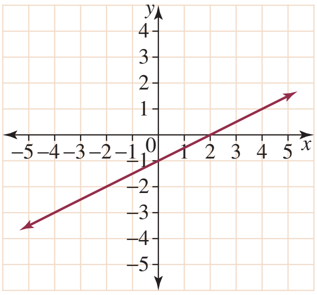


5 Calculate the gradient for each of the following straight line graphs.

a



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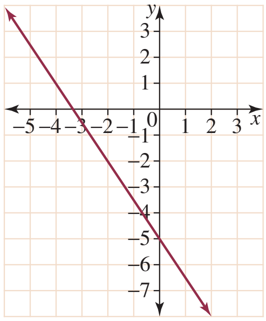
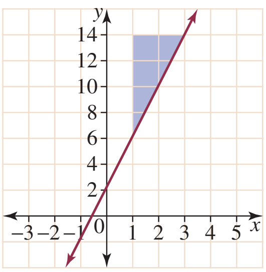


6 Use the triangles provided for each straight line graph below to determine the gradients of the lines. *Hint:* Check the scale on the axes before simply counting squares.

a



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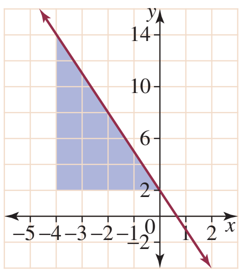
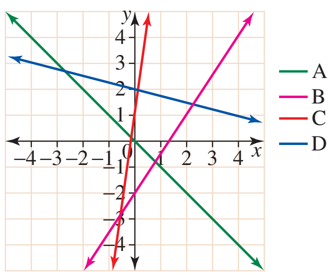


7 Copy and complete the following table.

	Rise	Run	Gradient
a	10 metres	4 metres	
b	10 metres		5
c		10 metres	$\frac{1}{2}$



8 **a** Match the descriptions given below with their corresponding line.

i Straight line with a *y*-intercept of (0, 1) and a positive gradient

ii Straight line with a gradient of $1\frac{1}{2}$

iii Straight line with a gradient of $^{-}1$

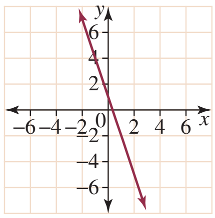
b Write a description for the unmatched graph.

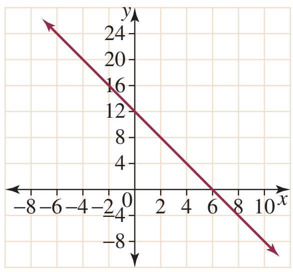
INDIVIDUAL PATHWAYS

eBook*plus*

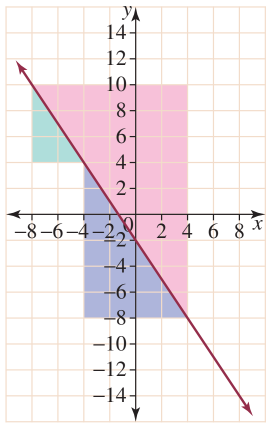
Digital docs:
Activity 3-B-1 Reviewing linear equations
Activity 3-B-2 Features of linear equations
Activity 3-B-3 Describing more complex linear equations

9 EXAMPLE 1 The graphs shown represent linear equations. Use the graphs to complete the following.
   a State the y-intercept for the line.
   b Determine the gradient of the line and describe the graph.

i


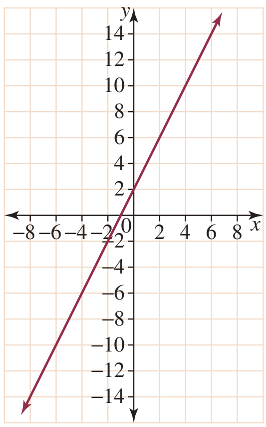
ii


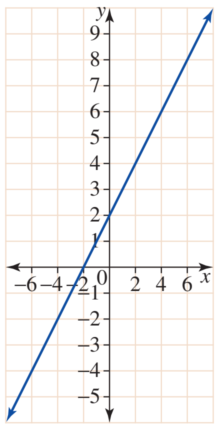
10 Three right-angled triangles have been superimposed on the graph at right.
   a Use each of these to determine the gradient of the line.
   b Does it matter which points are chosen to determine the gradient of a line? Explain.
   c Describe the shape of the graph.



Going further

Deceptive slopes





1 Without conducting calculations, which line shown (red or blue) appears to have the greatest gradient?
2 Calculate the gradient of both the red and blue lines.
3 Using your answer to part 2, which line has the greatest gradient? Explain your findings.

Extension

Curved lines are lines where the gradient is not constant. Straight lines can be used to approximate the gradient of curved lines.

1 On graph paper, draw a curved line on a Cartesian plane.
2 Choose two points on the curve. Connect them with a straight line and calculate the gradient of the straight line joining the two points.
3 Leave one point on the curve where it is, but move the second point closer to the first point. Calculate the gradient of the straight line joining these two points.
4 How could you use this method to calculate the gradient of a curve at a given point?

REFLECTION

WHOLE CLASS

Can you devise a simple diagram to help you remember when the slope is positive, negative or zero?