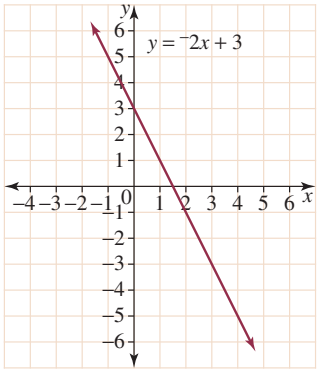
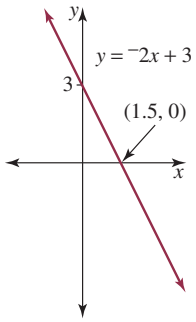
3-F Sketching linear graphs

KEY CONCEPTS

- A linear graph can be drawn as a plot or a sketch. For example, the equation $y = {}^{-}2x + 3$ is shown here as both a plot and a sketch.





- A sketch preserves all the defining features of the plot. For example, both the plot and the sketch of $y = {}^{-}2x + 3$ show that:
  - it is a straight line
  - it has a particular slope.
  - it passes through the x-axis at (1.5, 0)
  - it passes through the y-axis at (0, 3).

The table below outlines the similarities and differences of a plot and a sketch.

Category	Plot	Sketch
Cartesian plane	Drawn to scale	Not necessarily drawn to scale
Axes scale	Scale shown	Scale not shown
Determining line position	A number of points accurately plotted	Two points (minimum), labelled
y-intercept	Can be read from scale	Labelled
x-intercept	Can be read from scale	Labelled
Coordinates of other points	Can be read from axes	Can be estimated relative to labelled points (not recommended)
Gradient	Can be calculated from any plotted points	Can be calculated from given points

- To sketch a straight line, the positions of two points on the line need to be determined. These two points could be:
  - the x- and y-intercepts
  - any two points on the line
  - the y-intercept and another point determined using the gradient.
- Sketching linear equations involves 5 steps.
  1. Determining which two points to use
  2. Drawing and labelling the Cartesian plane

3. Marking the position of the first point
4. Marking the position of the second point
5. Drawing and labelling the line

The table below outlines the five steps involved in sketching a linear equation relative to the two points being used.

	Using *y*-intercept and gradient	Using *x*- and *y*-intercepts	Using two points
Step 1	Calculate m and c.	Calculate the x- and y-intercepts.	Calculate the coordinates of two points known to be on the line.
Step 2	Draw a Cartesian plane and label the axes (no scale required).	Draw a Cartesian plane and label the axes (no scale required).	Draw a Cartesian plane and label the axes (no scale required).
Step 3	On the y-axis, place a small dot for the y-intercept and label it appropriately.	On the y-axis, place a small dot for the y-intercept and label it appropriately.	Place a small dot for the first point and label it appropriately.
Step 4	From this point, estimate the rise and run according to m and mark in a second point with a dot. Label the point.	On the x-axis, place a small dot on the x-intercept relative to the y-intercept and label it appropriately.	Place a small dot for the second point relative to the first and label it appropriately.
Step 5	Draw in a straight line that passes through the two points.	Draw in a straight line that passes through the two points.	Draw in a straight line that passes through the two points.

Sketching using the *y*-intercept and the gradient

EXAMPLE 1

Sketch the graph of $y = -2x + 5$ using the y-intercept and the gradient.

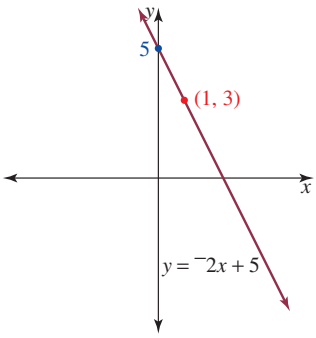
THINK

WRITE

1. Identify the y-intercept and the gradient from the equation $y = mx + c$.

$y = -2x + 5$
$m = -2$
$c = 5$

2. Draw and label a Cartesian plane.
3. Mark in the position of the y-intercept, as shown in blue. Label the point.
4. From the y-intercept, use the rise and run to mark the position of a second point.
   In this case $m = -2 = \frac{-2}{1} = \frac{\text{rise}}{\text{run}}$. From the y-intercept, rise -2 and run 1. Place a dot at (1, 3), as shown in red, and label the point.
5. Draw in a straight line that passes through the two points and label the line.



Sketching using the *x*- and *y*-intercepts

EXAMPLE 2

Sketch the linear equation $y = \frac{2}{3}x - 2$, using the x- and y-intercepts.

THINK

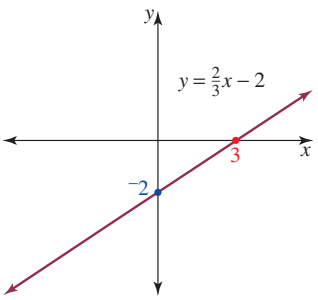
WRITE

1. Use the equation to determine the x- and y-intercepts.
   - The y-intercept = c in the equation $y = mx + c$.
   - The x-intercept occurs when $y = 0$. Substitute $y = 0$ and solve for x by adding 2 to both sides, multiplying both sides by 3 and dividing both sides by 2.

→ $y = \frac{2}{3}x - 2$
$c = {}^{-}2$
The y-intercept is the point $(0, {}^{-}2)$.
To find the x-intercept, let $y = 0$.
$y = \frac{2}{3}x - 2$
$0 = \frac{2}{3}x - 2$
$2 = \frac{2}{3}x$
$6 = 2x$
$x = 3$
The x-intercept is the point $(3, 0)$.

2. Draw and label a Cartesian plane.
3. Mark the position of the y-intercept, as shown in blue. Label the point.
4. Mark the x-intercept on the x-axis with a small dot, as shown in red, and label the point.
5. Draw a straight line that passes through the two points and label the line.

→



Sketching using two points

Sometimes when sketching using the x- and y-intercepts, both intercepts are at the origin. In this case, choose another x-value and substitute it into the equation to calculate the corresponding y-coordinate.

EXAMPLE 3

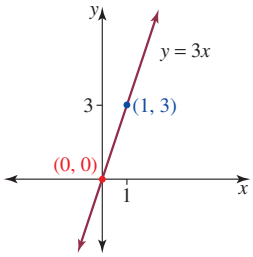
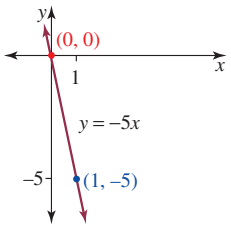
Sketch the graphs of: **a** $y = 3x$ **b** $y = {}^{-}5x$.

THINK

WRITE

a 1. The x-intercept is found by substituting $y = 0$ into the equation.

→ To find the x-intercept, let $y = 0$.
$y = 3x$
$0 = 3x$
$0 = x$
The x-intercept is $(0, 0)$.

THINK	WRITE
2 The y-intercept is found by substituting $x = 0$ into the equation.	To find the y-intercept, let $x = 0$. $y = 3x$ $y = 3 \times 0$ $y = 0$ The y-intercept is (0, 0).
3 As both intercepts are at the origin, another point is needed to sketch the line. Choose another x-value, and substitute it into the equation.	Let $x = 1$. $y = 3x$ $y = 3 \times 1$ $y = 3$ Another point is (1, 3).
4 • Draw and label a Cartesian plane. • Mark in the two points and label them. • Draw a straight line through the two points and label the line.	
b 1 The x-intercept is found by substituting $y = 0$ into the equation.	Let $y = 0$. $0 = {}^{-}5x$ $0 = x$ The x-intercept is (0, 0).
2 Since the x-intercept is (0, 0), and this is also the y-intercept, another point is needed to sketch the line. Choose another x-value and substitute it into the equation.	Let $x = 1$. $y = {}^{-}5x$ $y = {}^{-}5(1)$ $y = {}^{-}5$
3 Use the two points to draw and label a sketch of the graph.	

LEARNING EXPERIENCE

WHOLE CLASS

Which is which?

Equipment: BLM doc-0111 *Which is which?*

1 In pairs, match the equations with the appropriate graph on the worksheet.

2 Match the pairs of points with the appropriate graph.

3 One linear graph has no 'equation' or 'pair of points' description. Write an equation and choose two points that could be used to describe the line.

4 As a class, discuss the strategies you used to match the graph, equation and points.

EXERCISE 3F

Now try these

1 What is the difference between plotting a graph and sketching a graph?

2 Plot the following pairs of points and draw the line that passes through them.

a $(^{-}1, 3)$ and $(2, ^{-}2)$ **b** $(0, 3)$ and $(4, 0)$

c $(0, ^{-}4)$ and $(1, 0)$ **d** $(^{-}7, 8)$ and $(0, 1)$

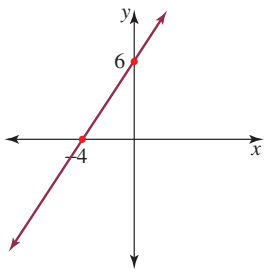
3 Draw a flow chart to summarise the steps taken in sketching a linear equation.

4 EXAMPLE 1 Sketch the following equations using the y-intercept and the gradient.

a $y = 3x - 10$ **b** $y = \frac{^{-}2}{5}x - 1$

5 MULTIPLE CHOICE Which sketch below represents the equation $y = \frac{3}{2}x - 6$?

A



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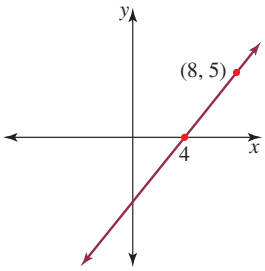
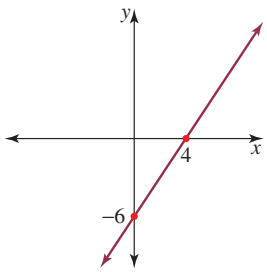
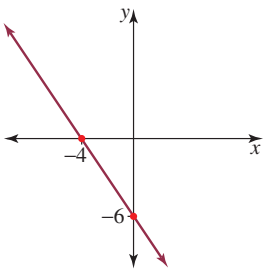


C



D



6 Sketch the graph of the linear function with a gradient of 2.5 and a y-intercept of 4.

7 Sketch the graph of the linear function with a gradient and y-intercept of 4.

8 EXAMPLE 2 Sketch the following equations using their x- and y-intercepts.

a $y = ^{-}4x + 12$ **b** $y = \frac{1}{2}x - 3$

9 EXAMPLE 3 Sketch the graph of $y = ^{-}2x$.

10 For the equation $y = ^{-}4x - 2$:

a calculate the y-value when $x = ^{-}3$

b calculate the y-value when $x = 2$

c sketch the equation using the points found in parts a and b.

11 Sketch the following linear equations.

a $y = \frac{3}{4}x - 4$ **b** $y = ^{-}5x + 1$ **c** $y = \frac{^{-}2}{5}x + 7$

12 Sketch the following linear equations on the same Cartesian plane showing x- and y-intercepts.

a $y = 2x + 12$ **b** $y = \frac{^{-}1}{2}x + 6$ **c** $y = x - 1.5$

INDIVIDUAL PATHWAYS

eBookplus

Digital docs:

Activity 3-F-1
Understanding linear graph sketching

Activity 3-F-2
Sketching linear graphs

Activity 3-F-3
Sketching tricky linear graphs

13 a Sketch the linear equation $y = \frac{-5}{7}x - \frac{3}{4}$:

- **i** using the y-intercept and the gradient
- **ii** using the x- and y-intercepts
- **iii** using two other points.

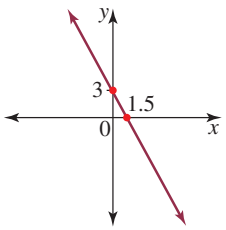
b Compare and contrast the methods and generate a list of advantages and disadvantages for each method. Which method do you think is best? Why?

Going further

What's the equation for this?

MULTIPLE CHOICE Which of the sketches below does not represent the equation $y = -2x + 3$?

A



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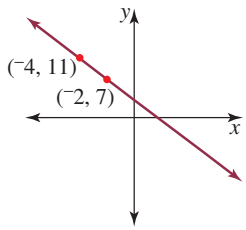

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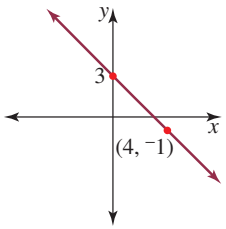

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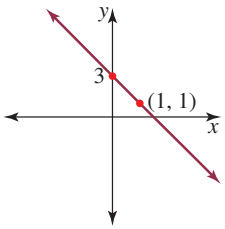
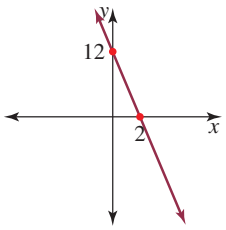


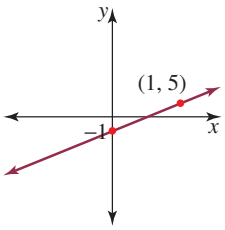
Extension

It is possible to use two points to determine the equation of a straight line through those points. Determine the equations that represent the sketches below.

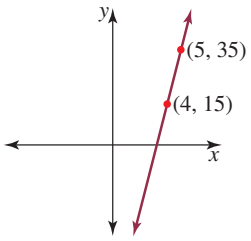
1



2



3



REFLECTION

WHOLE CLASS

What is the best method to use to sketch a linear graph?