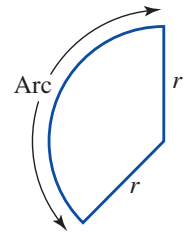


12C Area and perimeter of a sector

- A **sector** is a part of a circle formed by two radii and the section of circumference between them.
- The curved part of the sector is called an **arc**.
- If the sector is a specific fraction of an entire circle (e.g., one-quarter), that fraction is used to calculate the area.



WORKED EXAMPLE 9

Calculate the area enclosed by the figure at right, correct to 1 decimal place.

THINK

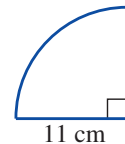
- 1 The given shape is a quadrant and thus its area can be found by dividing the area of a circle by 4. Write the appropriate formula.
- 2 State the value of the pronumeral.
- 3 Substitute the value of the pronumeral into the formula.
- 4 Evaluate and round the answer correct to 1 decimal place. Include the appropriate units.

WRITE

$$A_{\text{quadrant}} = \frac{1}{4} \pi r^2$$

$$r = 11$$

$$A = \frac{1}{4} \times \pi \times 11^2 \\ \approx 95.0 \text{ cm}^2$$



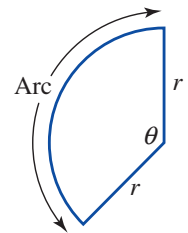
The general formula for the area of a sector

- Usually sectors are specified by the angle (θ) between the two radii.
- If $\theta = 360^\circ$, the sector would be an entire circle and its area would be:

$$\pi r^2 \text{ or } \frac{360}{360} \times \pi r^2.$$

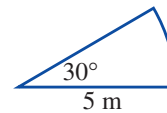
- For any other value of θ , then, the area of the sector is given by:

$$\text{Area} = \frac{\theta}{360} \times \pi r^2.$$



WORKED EXAMPLE 10

Calculate the area of the sector shown, correct to 1 decimal place.



THINK

- 1 Write the formula for the area of the sector.
- 2 Identify the values of the pronumerals.
- 3 Substitute the values of the pronumerals into the formula.
- 4 Evaluate and round the answer correct to 1 decimal place. Include the appropriate units.

WRITE

$$A = \frac{\theta}{360} \pi r^2$$

$$\theta = 30^\circ, r = 5$$

$$A = \frac{30}{360} \times \pi \times 5^2$$

$$\approx 6.5 \text{ m}^2$$

The perimeter of a sector

- If $\theta = 360^\circ$, the sector would be an entire circle, and its perimeter (circumference) would be:

$$2\pi r \text{ or } \frac{360}{360} \times 2\pi r.$$

- For any other value of θ , the length of the *curved* part (arc) of the sector is given by:

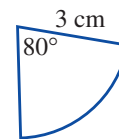
$$l = \frac{\theta}{360} \times 2\pi r.$$

- For the total perimeter of a sector, add the length of the two radii (r), and the length of the arc so that:

$$P = \frac{\theta}{360} \times 2\pi r + 2r.$$

WORKED EXAMPLE 11

Calculate the perimeter of the sector shown, correct to 1 decimal place.



THINK

- 1 Write the formula for the length of the curved side.
- 2 Identify the values of the pronumerals.
- 3 Substitute the values of the pronumerals into the formula.
- 4 Evaluate and round the answer correct to 1 decimal place.
- 5 Add all the sides together to calculate the perimeter. Include the appropriate units.

WRITE

$$l = \frac{\theta}{360} 2\pi r$$

$$\theta = 80^\circ, r = 3$$

$$l = \frac{80}{360} \times 2 \times \pi \times 3$$

$$\approx 4.2 \text{ cm}$$

$$P = 4.2 + 3 + 3$$

$$= 10.2 \text{ cm}$$

REMEMBER

1. A sector is a portion of a circle formed by two radii and the arc between them.
2. To find the area of a sector, use the formula $A = \frac{\theta}{360} \pi r^2$, where θ is the angle included between the radii.
3. To find the perimeter of a sector, use the formula $l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle included between the radii, to find the curved side. Then add the lengths of all sides to find the total perimeter.

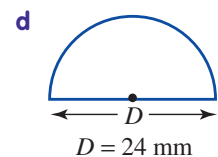
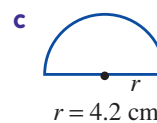
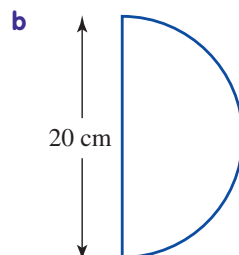
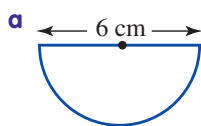
EXERCISE

12C

Area and perimeter of a sector

FLUENCY

- 1 Calculate the area of the semicircles below, correct to 2 decimal places.



INDIVIDUAL PATHWAYS

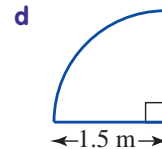
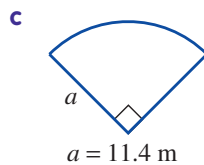
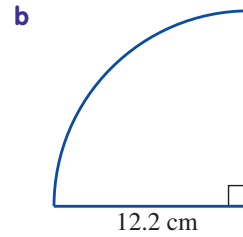
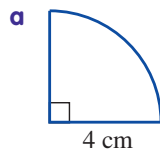
eBook *plus*

Activity 12-C-1
Exploring perimeter and area of sectors
doc-4148

Activity 12-C-2
Calculating perimeter and area of sectors
doc-4149

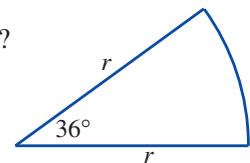
Activity 12-C-3
Applications of perimeter and area of sectors
doc-4150

- 2 **WE9** For each of the quadrants shown, calculate to 1 decimal place:
i the perimeter
ii the area enclosed.

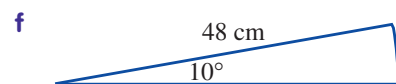
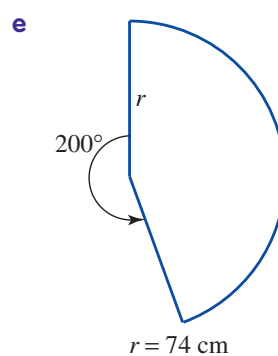
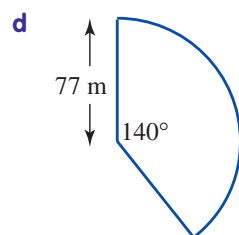
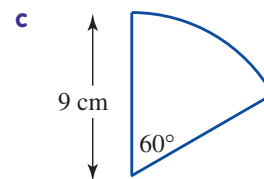
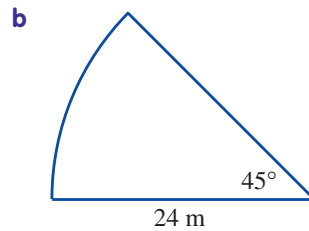
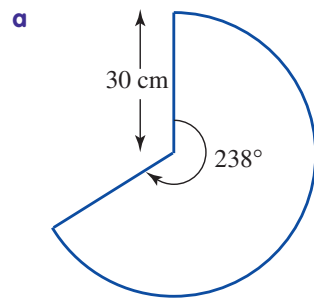


- 3 **MC** Which is the correct formula for calculating the area of this sector?

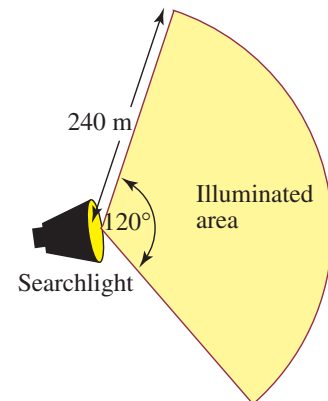
- A $A = \frac{1}{4}\pi r^2$ B $A = \frac{3}{4}\pi r^2$
C $A = \frac{1}{2}\pi r^2$ D $A = \frac{1}{10}\pi r^2$
E $A = \frac{1}{12}\pi r^2$



- 4 **WE10, 11** For each of the sectors shown, calculate to 1 decimal place:
i the perimeter ii the area.

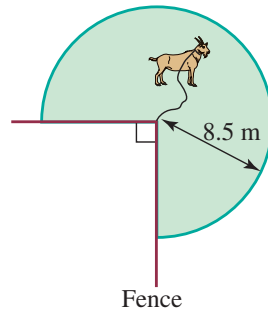
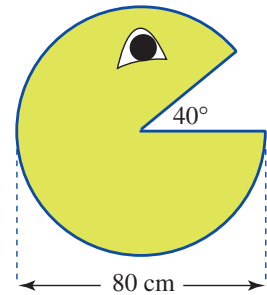


- 5 A searchlight lights up the ground to a distance of 240 m. What area does the searchlight illuminate if it can swing through an angle of 120° , as shown in the diagram at right? (Give your answer correct to 1 decimal place.)

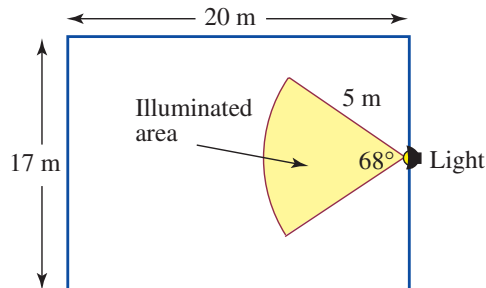


UNDERSTANDING

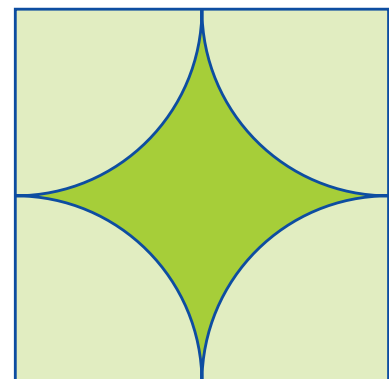
- 6 Calculate the perimeter, correct to 1 decimal place, of the cartoon figure at right.
- 7 A goat is tethered by an 8.5 m rope, to the outside of a corner post in a paddock, as shown in the diagram below. Calculate the area of grass (shaded) on which the goat is able to graze. (Give your answer correct to 1 decimal place.)



- 8 A beam of light is projected onto a theatre stage as shown in the diagram.

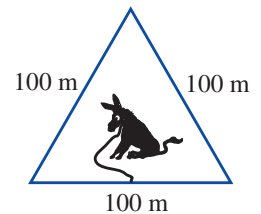
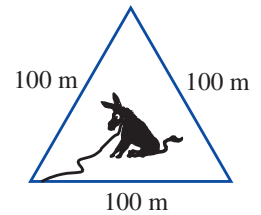
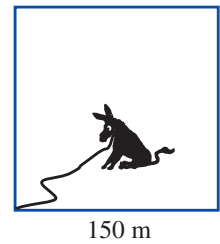


- a Calculate the illuminated area (correct to 1 decimal place) by finding the area of the sector.
- b Calculate the percentage of the total stage area that is illuminated by the light beam.
- 9 **MC** A sector has an angle of 80° and a radius of 8 cm, while another sector has an angle of 160° and a radius of 4 cm. The ratio of the area first sector to the area of the second sector is:
- A 1 : 2
 B 2 : 1
 C 1 : 1
 D 1 : 4
 E 4 : 1
- 10 Calculate the radius of the following sectors.
- a Area = 100 m^2 , angle = 13°
 b Area = 100 m^2 , curved arc = 12 m
 c Perimeter = 100 m, angle = 11°
- 11 Four baseball fields are to be constructed inside a rectangular piece of land. Each field is in the shape of a sector of a circle, as shown in light green. The radius of each sector is 80 m.
- a Calculate the area of one baseball field.
 b What percentage of the total area is occupied by the four fields?
 c The purchase of the land is \$24 000 per hectare. What is the total purchase price of the land?

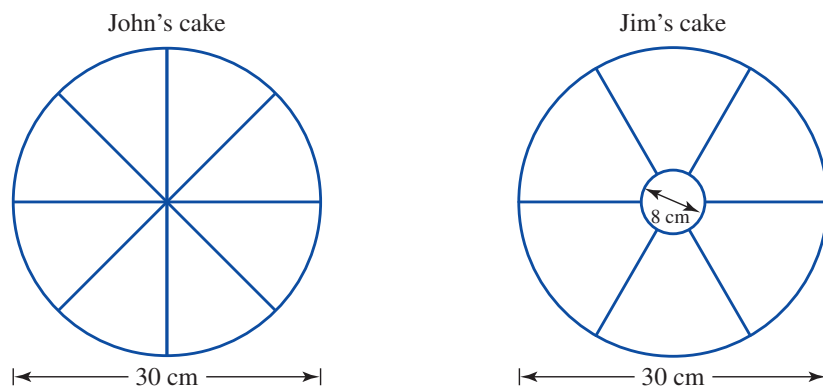


REASONING

- 12 a** A donkey inside a square enclosure is tethered to a post at one of the corners. Determine the length of the rope required so that the donkey eats only half of the grass in the enclosure.
- b** Suppose 2 donkeys are tethered at opposite corners of the square region shown at right. How long should the rope be so that the donkeys together can graze half of the area?
- c** This time four donkeys are tethered, one at each corner of the square region. How long should the rope be so that all the donkeys can graze only half of the area?
- d** Another donkey is tethered to a post inside an enclosure in the shape of an equilateral triangle. The post is at one of the vertices. Calculate how long the rope should be so that the donkey eats only half of the grass in the enclosure.
- e** This time the donkey is tethered halfway along one side of the equilateral triangular region shown at right. How long should the rope be so that the donkey can graze half of the area?



- 13** John and Jim are twins, and on their birthday they have two identical birthday cakes. Each cake has a diameter of 30 cm. Grandma Maureen cuts John's cake into 8 equal sectors. Grandma Mary cuts Jim's cake with a circle of diameter 8 cm in the centre and then 6 equal portions from the rest.



- a** At the centre of John's cake, what angle does each sector make?
- b** What area of cake is a slice of John's cake?
- c** What area of cake is the small central circular part of Jim's cake?
- d** What area of cake is the larger section of Jim's cake?
- e** If each boy eats one slice of the largest part of their own cake, who eats the most cake?
- 14** A lighthouse has a light beam in the shape of a sector of a circle that rotates at 10 revolutions per minute and covers an angle of 5° at all times. A person stands 200 m from the lighthouse and is spotted by the beam. How long after he was last illuminated does he have until the beam comes around and spots him again?

REFLECTION

What is the relationship between the curved arc of a sector and the area of the sector?